Name: _____

1. (20 marks) Study the convergence of the series $\sum_{n=1}^{\infty}(\sqrt{n+1} - \sqrt{n})$.

   Solution 1.

   As

   $$s_n \equiv \sum_{j=1}^{n}(\sqrt{n+1} - \sqrt{n}) = \sqrt{n} - 1 \to \infty , \quad \to \infty ,$$

   we conclude that this series diverges. Recall that $\sum_{n=1}^{\infty} x_n$ converges implies that $\lim_{n\to\infty} x_n = 0$.

   Solution 2. We have

   $$\begin{aligned}
   x_n &\equiv \sqrt{n+1} - \sqrt{n} \\
   &= \frac{1}{\sqrt{n+1} + \sqrt{n}} \\
   &\geq \frac{1}{2\sqrt{n+1}} \\
   &\geq \frac{1}{2(n+1)} .
   \end{aligned}$$

   As $\sum_{n=1}^{\infty} \dfrac{1}{2(n+1)} = \infty$, by Comparison Test, $\sum_{n=1}^{\infty} x_n$ is divergent.

   Remark. Many of you tried Ratio Test, but it is of no use. In many cases, Comparison Test is better. Remember it.